1. Functionality

The AND-Gate takes a logical AND of phosphorelated OmpR and logical NOT YcgE. If phosphorelated OmpR is present tRNA is produced which is acetylated in another light-independent reaction. YcgE represses the transcription of T7RNA polymerase mRNA from the T7ptag gene. Since T7ptag gene has two amber mutations, only if both acetylated tRNA and the T7RNAP mRNA are present the mRNA can be translated into the protein. Hence only if YcgE concentration is low and OmpR-P concentration is high at the same time, T7RNA polymerase is produced and enables the expression of β-Galactosidase which results in the production of a black dye.

2. Equations

$$
\begin{aligned}
tRNA & \quad \dot{x}_1 = k_t \frac{\left(\frac{OmpR-P}{K1}\right)^2}{1+\left(\frac{OmpR-P}{K1}\right)^2} - (\gamma_1 + k_a)x_1 + \gamma_{2p}x_2 + 2k_{7p}x_3 \left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 \\
Aa-tRNA & \quad \dot{x}_2 = k_a x_1 - 2k_{7p}x_3 \left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 - \gamma_2 x_2 \\
T7RNAP_{mRNA} & \quad \dot{x}_3 = k_{7m}\left(1 - \frac{\left(\frac{YcgE}{K3}\right)^2}{1+\left(\frac{YcgE}{K3}\right)^2}\right) - \gamma_3 x_3 \\
T7RNAP & \quad \dot{x}_4 = k_{7p}x_3 \left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 - \gamma_4 x_4 \\
lacZ_{mRNA} & \quad \dot{x}_5 = \alpha_M\left(1 - \frac{\left(\frac{x_4}{K5}\right)^2}{1+\left(\frac{x_4}{K5}\right)^2}\right) - \gamma_M x_5 \\
\beta-Galactosidase & \quad \dot{x}_6 = \alpha_B x_5 - \gamma_B x_6 \\
dye & \quad \dot{x}_7 = \alpha_A x_6
\end{aligned}
$$

3. Parameters

Parameter	Value	Unit	Name	Source
k_t	$\frac{46.67}{60}$	$\frac{nM}{s}$	max transcription rate tRNA	PKU Beijing 2009
k_a	$\frac{0.08}{60}$	$\frac{1}{s}$	synthesis rate Aa-tRNA	PKU Beijing 2009
k_{7p}	$\frac{1.5625}{60}$	$\frac{nM}{s}$	max transcription rate T7RNAP	PKU Beijing 2009
k_{7m}	$\frac{268*0.05}{60}$	$\frac{1}{s}$	max translateion rate T7RNAP	PKU Beijing 2009
k_S	0.3	$\frac{1}{nM}$	AND Gate rate	PKU Beijing 2009
γ_0	1	-	threshold Aa-tRNA	guessed
γ_1	$\frac{1}{60*60}$	$\frac{1}{s}$	degradation of tRNA	PKU Beijing 2009
γ_2	$\frac{1}{40*60}$	$\frac{1}{s}$	degradation of Aa-tRNA	PKU Beijing 2009
γ_3	$\frac{1}{4.4*60}$	$\frac{1}{s}$	degradation of T7RNAP mRNA	PKU Beijing 2009
γ_4	$\frac{46.67}{40*60}$	$\frac{1}{s}$	degradation of T7RNAP	PKU Beijing 2009
$K1$	5	nM	response param. OmpR-P,tRNA	guessed
$K3$	600	nM	response param. YcgE,T7RNAP	guessed
$K5$	$\frac{k7p}{4*gamma}$	nM	response param T7RNAP,lacZ	guessed
α_M	$\frac{0.997}{60}$	$\frac{nM}{s}$	max transcription rate lacZ	Chaos Lac
α_B	$\frac{1.661e-5}{60}$	$\frac{1}{s}$	max translation rate lacZ	Chaos Lac
α_A	$\frac{20}{60}$	$\frac{1}{s}$	enzymatic reaction rate	Chaos Lac
γ_M	$\frac{0.411}{60}$	$\frac{1}{s}$	degradation lacZ mRNA	Chaos Lac
γ_B	$\frac{8.331e-4}{60}$	$\frac{1}{s}$	degradation β-Galactosidase	Chaos Lac

4. Initial Data

Name	Variable	Initial Value	Comment
$tRNA$	x_1	0	
$Aa-tRNA$	x_2	0	
$T7RNAP_{mRNA}$	x_3	0	
$T7RNAP$	x_4	0	
$lacZ_{mRNA}$	x_5	0	
$\beta - Galactosidase$	x_6	0	
dye	x_7	0	

5. Reference

The model for our AND-Gate is based on the model of the iGEM team PKU Beijing 2009 for their AND-Gate1. We modified the equations such that the change in tRNA and Aa-tRNA does not include the degradation of the mRNA which caused negativity of some concentrations in our model.

The Expression of lacZ is an adaption of the model given by "Dynamics and bistability in a reduced model of the lac operon"