CONTENTS

1. MODEL

The model for our AND-Gate is based on the model of the iGEM team PKU Beijing
2009 for their AND-Gate1. We modified the equations such that the change in tRNA
and Aa-tRNA does not include the degradation of the mRNA which caused negativity of
some concentrations in our model.[1]

For more details see the paper by Anderson [2].

The dye output was an adaption of the model proposed in the paper by Yildirim[3].

Logical AND-Gate
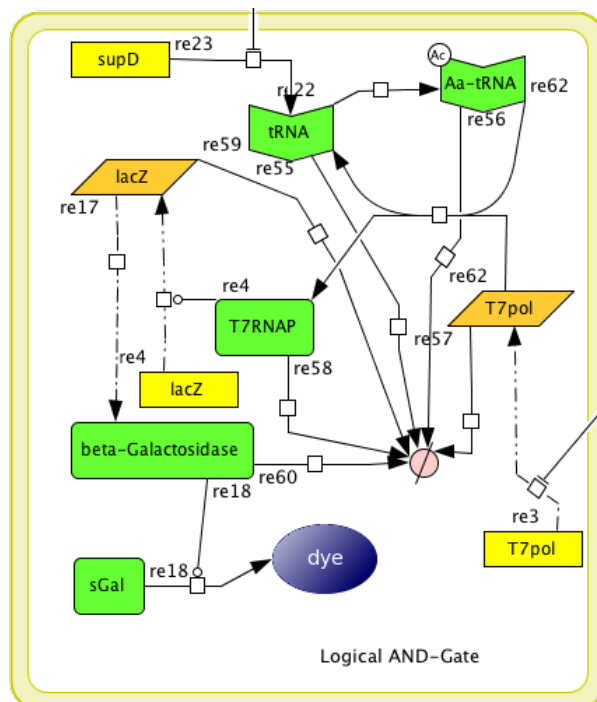
2. Equations

$$
\begin{array}{rcl}
tRNA & \dot{x}_1 = & k_t \dfrac{\left(\frac{OmpR-P}{K1}\right)^2}{1+\left(\frac{OmpR-P}{K1}\right)^2} - (\gamma_1 + k_a)x_1 + \gamma_{2p}x_2 + 2k_{7p}x_3\left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 \\[4ex]
Aa-tRNA & \dot{x}_2 = & k_a x_1 - 2k_{7p}x_3\left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 - \gamma_2 x_2 \\[4ex]
T7RNAP_{mRNA} & \dot{x}_3 = & k_{7m}\left(1 - \dfrac{\left(\frac{YcgE}{K3}\right)^2}{1+\left(\frac{YcgE}{K3}\right)^2}\right) - \gamma_3 x_3 \\[4ex]
T7RNAP & \dot{x}_4 = & k_{7p}x_3\left(\frac{\gamma_3}{k_{7m}}\right)\left(\frac{x_1}{\gamma_0+x_1}\right)^2 - \gamma_4 x_4 \\[4ex]
lacZ_{mRNA} & \dot{x}_5 = & \alpha_M\left(1 - \dfrac{\left(\frac{x_4}{K5}\right)^2}{1+\left(\frac{x_4}{K5}\right)^2}\right) - \gamma_M x_5 \\[4ex]
\beta-Galactosidase & \dot{x}_6 = & \alpha_B x_5 - \gamma_B x_6 \\[2ex]
dye & \dot{x}_7 = & \alpha_A x_6
\end{array}
$$

3. Parameters

Parameter	Value	Unit	Name	Source
k_t	$\frac{46.67}{60}$	$\frac{nM}{s}$	max transcription rate tRNA	[1]
k_a	$\frac{0.08}{60}$	$\frac{1}{s}$	synthesis rate Aa-tRNA	[1]
k_{7p}	$\frac{1.5625}{60}$	$\frac{nM}{s}$	max transcription rate T7RNAP	[1]
k_{7m}	$\frac{268*0.05}{60}$	$\frac{1}{s}$	max translation rate T7RNAP	[1]
k_S	0.3	$\frac{1}{nM}$	AND Gate rate	[1]
γ_0	1	-	threshold Aa-tRNA	guessed
γ_1	$\frac{1}{60*60}$	$\frac{1}{s}$	degradation of tRNA	[1]
γ_2	$\frac{1}{40*60}$	$\frac{1}{s}$	degradation of Aa-tRNA	[1]
γ_3	$\frac{1}{4.4*60}$	$\frac{1}{s}$	degradation of T7RNAP mRNA	[1]
γ_4	$\frac{46.67}{40*60}$	$\frac{1}{s}$	degradation of T7RNAP	[1]
$K1$	5	nM	response param. OmpR-P,tRNA	guessed
$K3$	600	nM	response param. YcgE,T7RNAP	guessed
$K5$	$\frac{k7p}{4*\gamma_4}$	nM	response param T7RNAP,lacZ	guessed
α_M	$\frac{0.997}{60}$	$\frac{nM}{s}$	max transcription rate lacZ	[3]
α_B	$\frac{1.661e-5}{60}$	$\frac{1}{s}$	max translation rate lacZ	[3]

Parameter	Value	Unit	Name	Source
α_A	$\frac{20}{60}$	$\frac{1}{s}$	enzymatic reaction rate	[3]
γ_M	$\frac{0.411}{60}$	$\frac{1}{s}$	degradation lacZ mRNA	[3]
γ_B	$\frac{8.331e-4}{60}$	$\frac{1}{s}$	degradation β-Galactosidase	[3]

4. Initial Data

Name	Variable	Initial Value	Comment	Source
$tRNA$	x_1	0		
$Aa - tRNA$	x_2	0		
$T7RNAP_{mRNA}$	x_3	0		
$T7RNAP$	x_4	0		
$lacZ_{mRNA}$	x_5	0		
$\beta - Galactosidase$	x_6	0		
dye	x_7	0		

5. Simulation

Since the AND-Gate is only an intermediate part, no additional simulation was done here.

References

1. PKU Beijing 2009, *And gate 1*, 2009.
2. J Christopher Anderson, Christopher A Voigt, and Adam P Arkin, *Environmental signal integration by a modular and gate*, Mol Syst Biol **3** (2007).
3. N Yildirim, M Santillan, D Horike, and MC Mackey, *Dynamics and bistability in a reduced model of the lac operon*, CHAOS **14** (2004), no. 2, 279–292 (English).